



1.4.3 Karnaugh Maps

Karnaugh Maps

A **Karnaugh map** is a grid used to simplify a Boolean expression from a **truth table**.

The values from the truth table are placed into the K-map, and groups of adjacent 1s are combined to find a simplified Boolean expression.

How to group the 1s

To simplify a Boolean expression:

- Put the values from a truth table into the K-map.
- Group all the 1s into boxes.
- The number of 1s in a box must be a power of 2. (1, 2, 4, 8, etc)
- The box must be as large as possible.
- The boxes can overlap.

NOTE: Boxes can go around the edge.

Example

Simplify the Boolean expression using a Karnaugh map.

$$(\neg A \wedge \neg B) \vee (A \wedge \neg B) \vee (\neg A \wedge B \wedge \neg C)$$

Put the values in a K-map by placing 1s in the cells that match the expression.

		AB			
		00	01	11	10
CD	00	1	1	0	1
	01	1	1	0	1
	11	1	0	0	1
	10	1	0	0	1





Group the 1s into boxes.

For this K-map, a box of eight 1s can be made by wrapping the box around the edge (boxed in red). The two remaining 1s can be put into a box of four by overlapping the boxes (boxed in blue).

		AB			
		00	01	11	10
CD	00	1	1	0	1
	01	1	1	0	1
	11	1	0	0	1
	10	1	0	0	1

Look for the variables that stay the same within a box.

In the red box in this K-map, B is always 0 and all the other variables change.

Therefore, this box can simplify to:

$$\neg B$$

In the blue box in this K-map, A and C are always 0 and all the other variables change.

Therefore, this box can simplify to:

$$\neg A \wedge \neg C$$

Lastly we join the expressions using OR to give the simplified expression:

$$\neg B \vee (\neg A \wedge \neg C)$$

