



1.4.3 Boolean Expressions

Boolean Logic

Boolean logic uses only two possible values:

- 1 = True
- 0 = False

Boolean expressions use logical operators to produce an output of 0 or 1.

Boolean Operators

Operator	Symbol	Meaning
AND	$A \wedge B$	1 if both inputs are 1
OR	$A \vee B$	1 if at least one input is 1
NOT	$\neg A$	Inverts the input
XOR	$A \underline{\vee} B$	1 if the inputs are different

Boolean Laws

Identities	
$A \wedge 0 = 0$	$A \vee 0 = A$
$A \wedge 1 = A$	$A \vee 1 = 1$
$A \wedge A = A$	$A \vee A = A$
$A \wedge \neg A = 0$	$A \vee \neg A = 1$
$\neg \neg A = A$	

Absorption law – absorb the brackets	
$A \vee (A \wedge B) = A$	$A \wedge (A \vee B) = A$

Commutative rules	
$A \vee B = B \vee A$	$A \wedge B = B \wedge A$

Distributive rules	
$A \wedge (B \vee C) = A \wedge B + A \wedge C$	$A \vee B \wedge C = (A \vee B) \wedge (A \vee C)$



**De Morgan's Laws – break the bar, change the sign**

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\neg(A \vee B) = \neg A \wedge \neg B$$

TIPS: Simplifying Boolean Expressions

When simplifying Boolean expressions, look for identities in the following order:

1) Look for 0s and 1s

$$A \wedge 0 = 0$$

$$A \vee 0 = A$$

$$A \wedge 1 = A$$

$$A \vee 1 = 1$$

2) Look for these identities

$$A \wedge A = A$$

$$A \vee A = A$$

$$A \wedge \neg A = 0$$

$$A \vee \neg A = 1$$

3) Look for absorption law

$$A \vee (A \wedge B) = A$$

$$A \wedge (A \vee B) = A$$

4) Multiply out brackets or factorise

$$A \wedge (B \vee C) = A \wedge B \vee A \wedge C$$

$$A \vee B \wedge C = (A \vee B) \wedge (A \vee C)$$

5) De Morgan's law

$$\neg(A \wedge B) = \neg A \vee \neg B$$

$$\neg(A \vee B) = \neg A \wedge \neg B$$

Following this will tend to make the expression reduce to smaller, more manageable expressions sooner.

