4.6.5 Boolean Algebra

NOTE: Only one example is shown for each answer, but multiple possible methods exist.

1) Using the rules of Boolean algebra, simplify the following Boolean expression.

$$\overline{\left(\overline{A + A \cdot (A + B)}\right) + \left(\overline{B} \cdot \overline{C}\right)}$$

You **must** show your working

[4 marks]

- $\overline{\left(\overline{A} + A\right) + \left(\overline{B} \cdot \overline{C}\right)}$ **By absorption $A = A \cdot (A + B)$**
- $\overline{\overline{1} + \left(\overline{B} \cdot \overline{C}\right)}$ **By identity $\overline{A} + A = 1$**
- $\overline{0 + \left(\overline{B} \cdot \overline{C}\right)}$ **By identity $\overline{1} = 0$**
- $\overline{\left(\overline{B} \cdot \overline{C}\right)}$ **By identity $0 + X = X$**
- $B + C$ **De Morgan's**

2) Using the laws of Boolean algebra, show that:

$$(A + B) \cdot (B + C \cdot (D + \overline{D})) = A \cdot C + B$$

You **must** show your working

[4 marks]

- $(A + B) \cdot (B + C \cdot 1)$ **By identity $D + \overline{D} = 1$**
- $(A + B) \cdot (B + C)$ **By identity $C \cdot 1 = C$**
- $A \cdot B + A \cdot C + B \cdot B + B \cdot C$ **Using distribution law**
- $A \cdot B + A \cdot C + B + B \cdot C$ **By identity $B \cdot B = B$**
- $A \cdot B + A \cdot C + B$ **By absorption $B + B \cdot C = B$**
- $A \cdot C + B$ **By absorption $A \cdot B + B = B$**

3) Identities are often applied to help simplify Boolean expressions. One such identity is:

$$A \cdot \overline{A} = 0$$

Without using a truth table, explain why this identity is true.

[2 marks]

- **If input A is 0 then NOT A will be 1 and if A is 1 then NOT A will be 0**
- **An AND gate only outputs 1 if both inputs are 1**

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4) Using the laws of Boolean algebra, simplify the following Boolean expression.

$$\overline{\overline{\overline{B} \cdot A} \cdot \overline{B}} + A \cdot B$$

You **must** show your working

[4 marks]

- $\overline{(\mathbf{B} + \overline{\mathbf{A}}) \cdot \overline{\boldsymbol{B}}} + \boldsymbol{A} \cdot \boldsymbol{B}$ **De Morgan's**
- $\overline{\mathbf{B} \cdot \overline{\mathbf{B}} + \overline{\mathbf{A}} \cdot \overline{\boldsymbol{B}}} + \boldsymbol{A} \cdot \boldsymbol{B}$ **Expand brackets**
- $\overline{\mathbf{0} + \overline{\mathbf{A}} \cdot \overline{\boldsymbol{B}}} + \boldsymbol{A} \cdot \boldsymbol{B}$ **By identity $\boldsymbol{B} \cdot \overline{\boldsymbol{B}} = \mathbf{0}$**
- $\overline{\overline{\mathbf{A}} \cdot \overline{\boldsymbol{B}}} + \boldsymbol{A} \cdot \boldsymbol{B}$ **By identity $\mathbf{0} \cdot \mathbf{X} = \mathbf{0}$**
- $\mathbf{A} + \mathbf{B} + \boldsymbol{A} \cdot \boldsymbol{B}$ **De Morgan's**
- $\mathbf{A} + \mathbf{B}$ **Absorption law $\boldsymbol{A} + \boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{A}$**

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5) Using the laws of Boolean algebra, simplify the following Boolean expression.

$$\overline{\overline{A\cdot(A+1)}\cdot\overline{B}}\cdot\overline{\overline{A}+\overline{B+0}}$$

You **must** show your working

[4 marks]

- $\overline{\overline{A\cdot 1}\cdot\overline{B}}\cdot\overline{\overline{A}+\overline{B+0}}$ — By identity $A+1 = 1$
- $\overline{\overline{A}\cdot\overline{B}}\cdot\overline{\overline{A}+\overline{B+0}}$ — By identity $A\cdot 1 = A$
- $\overline{\overline{A}\cdot\overline{B}}\cdot\overline{\overline{A}+\overline{B}}$ — By identity $B+0 = B$
- $\overline{\overline{A}\cdot\overline{B}+\overline{A}+\overline{B}}$ — De Morgan's $\overline{A}\cdot\overline{B}+\overline{A} = \overline{A}$
- $\overline{\overline{A}+\overline{B}}$ — Absorption law $\overline{A}+\overline{A}\cdot\overline{B}=\overline{A}$
- $A\cdot B$ — De Morgan's

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6)

a) Complete the truth table in **Figure 1** for the inputs A and B.

Figure 1

A	B	$A+B$	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$\overline{\overline{A}\cdot\overline{B}}$
0	0	0	1	1	1	0
0	1	1	1	0	0	1
1	0	1	0	1	0	1
1	1	1	0	0	0	1

[1 mark]

b) What law in Boolean algebra does this demonstrate?

[1 mark]

- De Morgan's law

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7) Using the laws of Boolean algebra, simplify the following Boolean expression.

$$\overline{\overline{A} + B \cdot C + B \cdot \overline{C}} + C \cdot (A + \overline{A} \cdot (B + 1))$$

You **must** show your working

[4 marks]

- $\overline{\overline{A} + B \cdot C + B \cdot \overline{C}} + C \cdot (A + \overline{A} \cdot 1)$ **By identity $B + 1 = 1$**
- $\overline{\overline{A} + B \cdot C + B \cdot \overline{C}} + C \cdot (A + \overline{A})$ **By Identity $A \cdot 1 = A$**
- $\overline{\overline{A} + B \cdot C + B \cdot \overline{C}} + C \cdot 1$ **By identity $A + \overline{A} = 1$**
- $\overline{\overline{A} + B \cdot C + B \cdot \overline{C}} + C$ **By Identity $C \cdot 1 = C$**
- $\overline{\overline{A} + B \cdot (C + \overline{C})} + C$ **Put into brackets**
- $\overline{\overline{A} + B \cdot 1} + C$ **By identity $C + \overline{C} = 1$**
- $\overline{\overline{A} + B} + C$ **By Identity $B \cdot 1 = B$**
- $A \cdot \overline{B} + C$ **De Morgan's law**

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8) Using the laws of Boolean algebra, simplify the following Boolean expression.

$$A \cdot \overline{B} + B \cdot \overline{A + (\overline{B} \cdot C)}$$

You **must** show your working

[4 marks]

- $A \cdot \overline{B} + B \cdot A \cdot \overline{\overline{B} \cdot C}$ **De Morgan's law**
- $A \cdot \overline{B} + B \cdot A \cdot (B + \overline{C})$ **De Morgan's law**
- $A \cdot (\overline{B} + B \cdot (B + \overline{C}))$ **Factorise out the A**
- $A \cdot (\overline{B} + B)$ **Absorption law $B \cdot (B + \overline{C}) = B$**
- $A \cdot 1$ **By identity $B + \overline{B} = 1$**
- A **By Identity $A \cdot 1 = A$**

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9) Using the rules of Boolean algebra, simplify the following Boolean expression.

$$\overline{A} \cdot (B \cdot C \cdot D + B \cdot C \cdot \overline{D} + B) + \overline{\overline{A} + B}$$

You **must** show your working

[4 marks]

- $\overline{A} \cdot (B \cdot C \cdot (D + \overline{D}) + B) + \overline{\overline{A} + B}$ — By factorising out $B \cdot C$
- $\overline{A} \cdot (B \cdot C \cdot 1 + B) + \overline{\overline{A} + B}$ — By identity $D + \overline{D} = 1$
- $\overline{A} \cdot (B \cdot C + B) + \overline{\overline{A} + B}$ — By Identity $C \cdot 1 = C$
- $\overline{A} \cdot B + \overline{\overline{A} + B}$ — Absorption law $B \cdot C + B = B$
- $\overline{A} \cdot B + A \cdot \overline{B}$ — de Morgan's law
- $A \oplus B$ — Simplified to XOR

Accept $\overline{A} \cdot B + A \cdot \overline{B}$ as final answer

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10) **Table 1** lists six Boolean equations. Three of them are correct, the others are not. Tick the cells next to the **three** equations are correct.

Table 1

Equation	Tick
$A \cdot \overline{A} = 1$	
$A + B = \overline{\overline{A} \cdot \overline{B}}$	✓
$A + 1 = 1$	✓
$A \cdot (A + B) = A$	✓
$A + (A \cdot B) = B$	
$A \cdot 1 = 1$	

[3 marks]

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11) Using the rules of Boolean algebra, simplify the following Boolean expression.

$$\overline{\overline{A} + \overline{B}} + B \cdot \overline{A}$$

You **must** show your working

[3 marks]

- $\mathbf{A \cdot B} + \boldsymbol{B} \cdot \overline{\boldsymbol{A}}$ **de Morgan's law**
- $\mathbf{B \cdot (A} + \overline{\boldsymbol{A}}\mathbf{)}$ **Factorise for B**
- $\boldsymbol{B} \cdot \mathbf{1}$ **By identity $\boldsymbol{A} + \overline{\boldsymbol{A}} = \mathbf{1}$**
- $\mathbf{B}$ **By Identity $\boldsymbol{B} \cdot \mathbf{1} = \boldsymbol{B}$**