4.5.2 Number Bases

The base of a number system tells us how many different digit symbols are available before the place value moves one column to the left.

Number base: Decimal

Decimal (base 10) is the number system most commonly used.

It contains the first ten digits from 0 to 9.

Each time we count up to 10 in a column we add 1 to the next column to the left.

1000s (10^3)	100s (10^2)	10s (10^1)	1s (10^0)
1	3	2	9

$$(1 \times 1000) + (3 \times 100) + (2 \times 10) + (9 \times 1)$$
$$= 1329$$

Number base: Binary

Binary (base 2) is very important in computer science because computers represent data using two states, such as on or off, true or false, or charge and no charge.

It contains just the digits 1 and 0.

Each time we count up to 2 in a column we add 1 to the next column to the left.

8s (2^3)	4s (2^2)	2s (2^1)	1s (2^0)
1	1	1	0

$$(1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$$
$$= 14$$

1110 in binary is 14 in decimal

Number base: Hexadecimal

Hexadecimal (base 16) is often used in computer science because it provides a more compact way to write long binary numbers.

For example, the binary number 1001011010101111 can be represented as the hexadecimal 96AF.

It contains the first ten digits 0–9 and the first six letters, where A = 10, B = 11, C = 12, D = 13, E = 14 and F = 15.

Each time we count up to 16 in a column we add 1 to the next column to the left.

4096s (16^3)	256s (16^2)	16s (16^1)	1s (16^0)
1	3	B	9

$$(1 \times 16^3) + (3 \times 16^2) + (11 \times 16^1) + (9 \times 16^0)$$

$$= 5049$$

$13B9$ in hexadecimal is 5049 in decimal

Decimal to Binary

Converting the decimal number 159 to binary.

- The highest power of 2 which we can subtract from 159 is $2^7(128)$
- We can then put a 1 in the 128 column.

128s	64s	32s	16s	8s	4s	2s	1s
1							

- Subtract 128 from 159. $159 - 128 = 31$
- The highest power of 2 which we can subtract from 31 is 2^4 (16).

128s	64s	32s	16s	8s	4s	2s	1s
1	0	0	1				

- Subtract 16 from 31. $31 - 16 = 15$.
- The highest power of 2 which we can subtract from 15 is 2^3 (8).

128s	64s	32s	16s	8s	4s	2s	1s
1	0	0	1	1			

- $15 - 8 = 7$. The highest power of 2 which we can subtract from 7 is 2^2 (4).

128s	64s	32s	16s	8s	4s	2s	1s
1	0	0	1	1	1		

- $7 - 4 = 3$. The highest power of 2 which we can subtract from 3 is 2^1 (2).

128s	64s	32s	16s	8s	4s	2s	1s
1	0	0	1	1	1	1	

- $3 - 2 = 1$.

128s	64s	32s	16s	8s	4s	2s	1s
1	0	0	1	1	1	1	1

The decimal value **159** in binary is **10011111**

Binary to Decimal

Converting the binary number 101011 to decimal.

- Put the binary number into a table with increasing powers of 2.
- Add up the values in the columns that contain a 1.

32s	16s	8s	4s	2s	1s
1	0	1	0	1	1

$$32 + 8 + 4 + 2 + 1 = 47$$

The binary value $\mathbf{101011}$ in decimal is $\mathbf{47}$

Binary to Hexadecimal

Converting the binary number 011011011111 to hexadecimal.

- To convert a binary number into hexadecimal we split the binary number into groups of 4 bits (a nibble).
- We treat each group of 4 bits as a separate number and convert each group into a hexadecimal digit.

$$011011011111$$

0110	1101	1111
6	D	F

011011011111 is $6DF$ in hexadecimal

Hexadecimal to Binary

Converting the hexadecimal number $4B7$ to binary.

- To convert a hexadecimal number, we take each digit and convert it into its 4-bit binary equivalent.
- We then combine these together.

4B7

4	B	7
0100	1011	0111

$6B7$ is 010010110111 in binary

Decimal to Hexadecimal

Converting the decimal value 293 to hexadecimal.

- 293 in binary is 100100101
- Split the binary number into groups of 4 bits and convert each group into a hexadecimal digit.

0001	0010	0101
1	2	5

The decimal value 293 is 125 in hexadecimal

Hexadecimal to Decimal

Converting the hexadecimal value A4E to decimal.

- A4E in binary is 101000101110
- Convert 101000101110 into decimal
- 2048 + 512 + 32 + 8 + 4 + 2 = 2606

The hexadecimal value A4E is 2606 in decimal