4.4.5 Turing Machines

1)

a) Describe the Halting problem.

[3 marks]

- **Determining if a program will halt;**
- **without running the program;**
- **for a particular input;**

b) Why is it not possible to create a Turing machine that solves the Halting problem?

[1 mark]

- **The Halting problem is non-computable / undecidable // there is no algorithm that solves the Halting problem;**
- **In general, inspection alone cannot always determine whether any given algorithm will halt for its given inputs // a program cannot be written that can determine whether any given algorithm will halt for its given inputs;**

c) To define a Turing machine the finite alphabet of symbols that it can use needs to be specified and there needs to be a tape.

State two other components of a Turing machine.

[2 marks]

- **Finite set of states (in a state transition diagram);**
- **A set of transition rules;**
- **A (sensing) read-write head (that can move along the tape one square at a time);**
- **Start state;**
- **Set of accepting / halting states;**
- **State register // current state;**

d) Explain what a Universal Turing Machine is.

[2 marks]

- **A Turing machine that can execute/simulate the behaviour of any other Turing machine;**
- **Faithfully executes operations on the data precisely as the simulated TM does;**
- **The description of/instructions for the TM and the TM's input are stored on the Universal Turing machine's tape;**

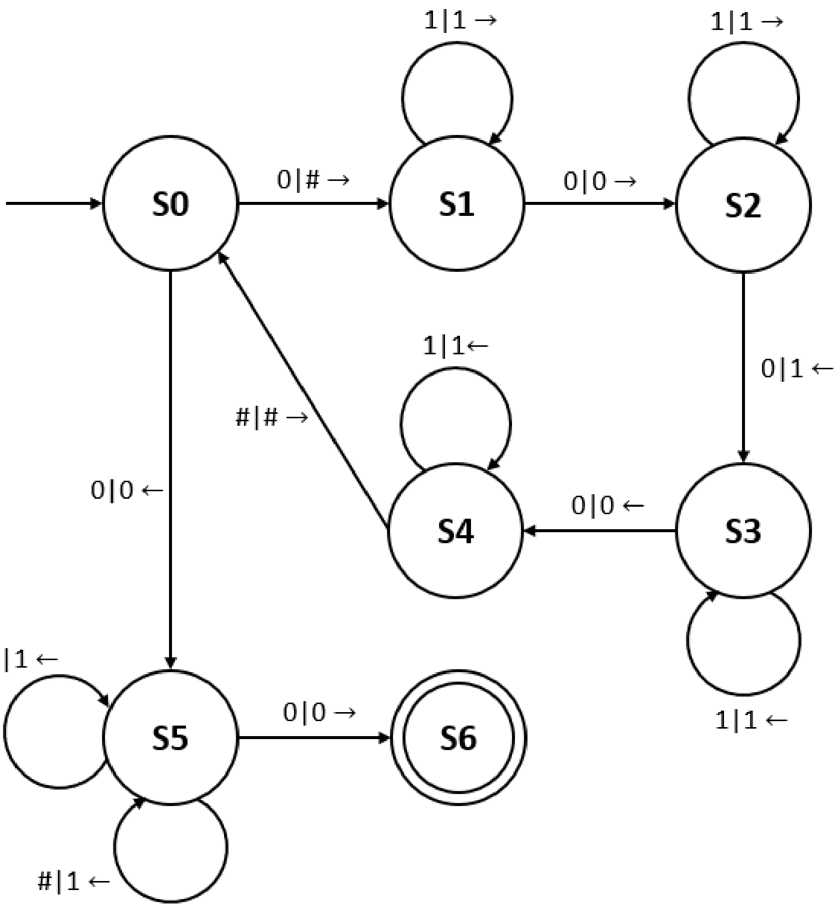
e) Why can a Universal Turing Machine be considered to be more powerful than any computer that you can purchase?

[1 mark]

- **Because it has an infinite amount of memory/tape;**

2) **Figure 1** shows the transition function (represented as a state transition diagram) and part of the tape for a Turing machine designed to complete a task. The current position of the read/write head is indicated by the * symbol.

Figure 1



...	0	1*	1	0	0	0	0	...

The label 1|#→ on the transition from S0 to S1 means if the machine is in state S0 and a 1 is read from the tape then a # should be written to the tape and then the read/write head moved one cell to the right.

a) After four steps of the computation have been completed, the current state, the tape contents and the position of the read/write head are:

Tape										Current State
...	0	1*	1	0	0	0	0	...		S3

Complete the unshaded cells of **Table 1** to show the result of tracing the computation of this Turing machine, from step five onwards. Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed. Step four has been repeated at the start of the trace.

Table 1

Tape										Current State
...	0	1*	1	0	0	0	0	...		S3
...	**0**	**#**	**1***	**0**	**1**	**0**	**0**	...		**S4**
...	**0**	**#***	**1**	**0**	**1**	**0**	**0**	...		**S4**
...	**0**	**#**	**1***	**0**	**1**	**0**	**0**	...		**S0**
...	**0**	**#**	**#**	**0***	**1**	**0**	**0**	...		**S1**
...	**0**	**#**	**#**	**0**	**1***	**0**	**0**	...		**S2**
...	**0**	**#**	**#**	**0**	**1**	**0***	**0**	...		**S2**
...	**0**	**#**	**#**	**0**	**1***	**1**	**0**	...		**S3**
...	**0**	**#**	**#**	**0***	**1**	**1**	**0**	...		**S3**
...	**0**	**#**	**#***	**0**	**1**	**1**	**0**	...		**S4**
...	**0**	**#**	**#**	**0***	**1**	**1**	**0**	...		**S0**
...	**0**	**#**	**#***	**0**	**1**	**1**	**0**	...		**S5**
...	**0**	**#***	**1**	**0**	**1**	**1**	**0**	...		**S5**
...	**0***	**1**	**1**	**0**	**1**	**1**	**0**	...		**S5**
...	**0**	**1***	**1**	**0**	**1**	**1**	**0**	...		**S6**

[5 marks]

b) What is the purpose of the Turing machine shown in **Figure 1**?

[1 mark]

- **Make a copy of a string of 1s;**

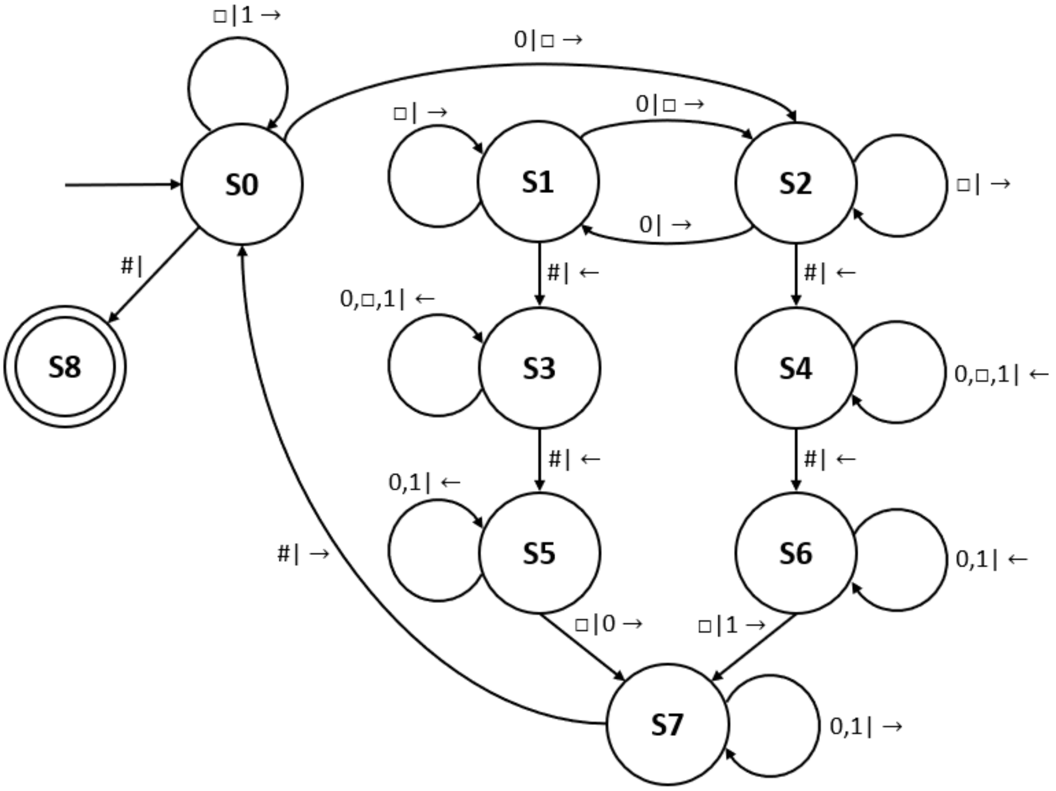
c) What is the purpose of the transition from S5 to S6 in the Turing machine in **Figure 1**?

[1 mark]

- **Moves the read/write head to the start of the (original) string of 1s // moves the read/write head back to where it started from;**

3) **Figure 2** shows the transition function (represented as a state transition diagram) and part of the tape for a Turing machine designed to count the number of zeros between two # symbols on the tape. The current position of the read/write head is indicated by the * symbol.

Figure 2



...	#	0*	0	0	#	...

The □ symbol is used to denote an empty cell on the tape. The label on the transition from S6 to S7 means if the machine is in state S6 and a □ is read from the tape, then a 0 should be written to the tape and then the read/write head is moved one cell to the right. The label on the transition from S3 to S3 means if the machine is in state S3 and a □, a 0 or a 1 is read from the tape, then the read/write head is moved one cell to the left; the contents of the tape are not changed.

a) Complete the unshaded cells of **Table 2** to show the result of tracing the computation of this Turing machine for the first 20 steps only. Show the contents of the tape, the current position of the read/write head and the current state as the input is processed. The top row of the table shows the initial state, the initial position of the read/write head and the starting contents of the tape.

Table 2

Tape									Current State
...			#	0*	0	0	#	...	S0
...			#		0*	0	#	...	S2
...			#		0	0*	#	...	S1
...			#		0		#*	...	S2
...			#		0	*	#	...	S4
...			#		0*		#	...	S4
...			#	*	0		#	...	S4
...			#*		0		#	...	S4
...		*	#		0		#	...	S5
...		1	#*		0		#	...	S7
...		1	#	*	0		#	...	S0
...		1	#		0*		#	...	S0
...		1	#			*	#	...	S2
...		1	#				#*	...	S2
...		1	#			*	#	...	S4
...		1	#		*		#	...	S4
...		1	#	*			#	...	S4
...		1	#*				#	...	S4
...		1*	#				#	...	S5
...	*	1	#				#	...	S5
...	1	1*	#				#	...	S7

[5 marks]

b) Describe two circumstances when the transition function for this Turing machine will not correctly count the number of zeros between two # symbols on the tape. You should assume that:

- the read/write head starts immediately to the right of a cell containing a #
- there is a cell containing a # symbol somewhere on the tape to the right of the read/write head's initial position
- there are exactly two # symbols on the tape.

[2 marks]

- **When there are no zeros between the two hash symbols on the tape;**
- **When there is a character other than zero (A. a one) between the two hash symbols on the tape;**
- **If the machine was not in the start state (S0);**