



4.4.5 Turing Machines

1)

a) Describe the Halting problem.

[3 marks]

b) Why is it not possible to create a Turing machine that solves the Halting problem?

[1 mark]

c) To define a Turing machine the finite alphabet of symbols that it can use needs to be specified and there needs to be a tape.

State two other components of a Turing machine.

[2 marks]

d) Explain what a Universal Turing Machine is.

[2 marks]

e) Why can a Universal Turing Machine be considered to be more powerful than any computer that you can purchase?

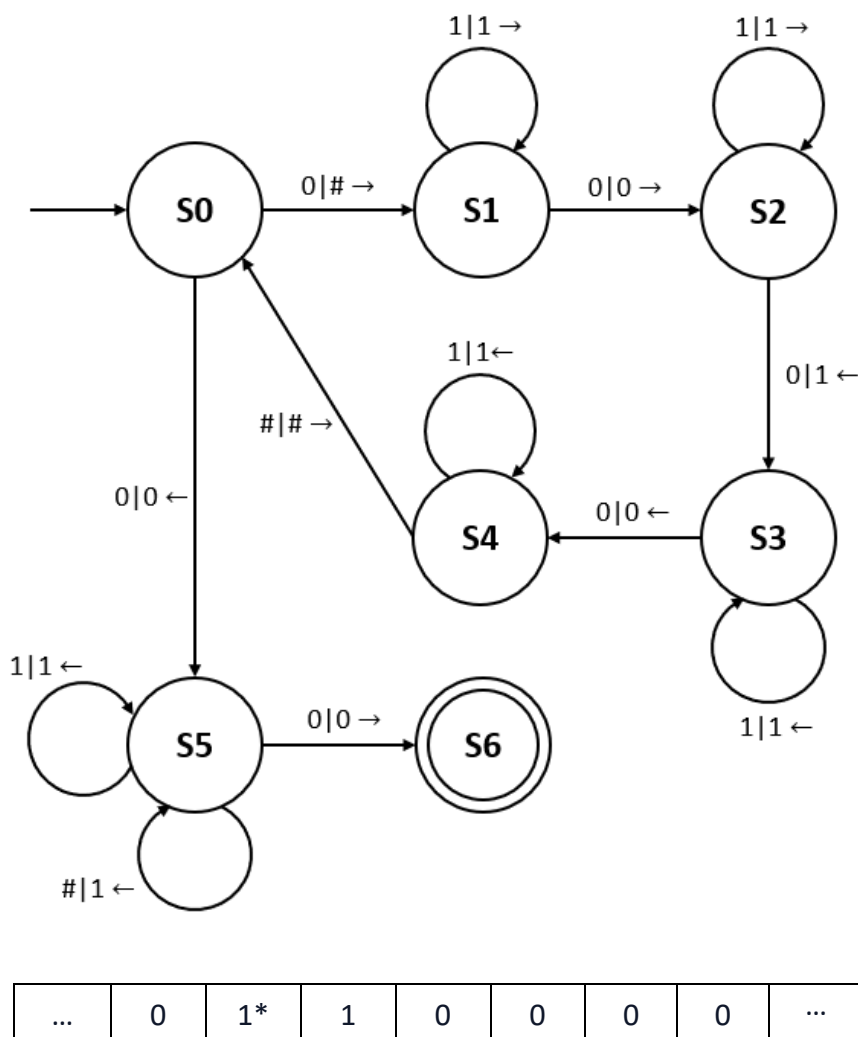
[1 mark]





- 2) **Figure 1** shows the transition function (represented as a state transition diagram) and part of the tape for a Turing machine designed to complete a task. The current position of the read/write head is indicated by the * symbol.

Figure 1



The label $1 | \# \rightarrow$ on the transition from S0 to S1 means if the machine is in state S0 and a 1 is read from the tape then a # should be written to the tape and then the read/write head moved one cell to the right.





- a) After four steps of the computation have been completed, the current state, the tape contents and the position of the read/write head are:

Tape									Current State
...	0	1*	1	0	0	0	0	...	S3

Complete the unshaded cells of **Table 1** to show the result of tracing the computation of this Turing machine, from step five onwards. Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed. Step four has been repeated at the start of the trace.

Table 1

Tape									Current State
...	0	1*	1	0	0	0	0	...	S3
...								...	
...								...	
...								...	
...								...	
...								...	
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[5 marks]

- b) What is the purpose of the Turing machine shown in **Figure 1**?

[1 mark]

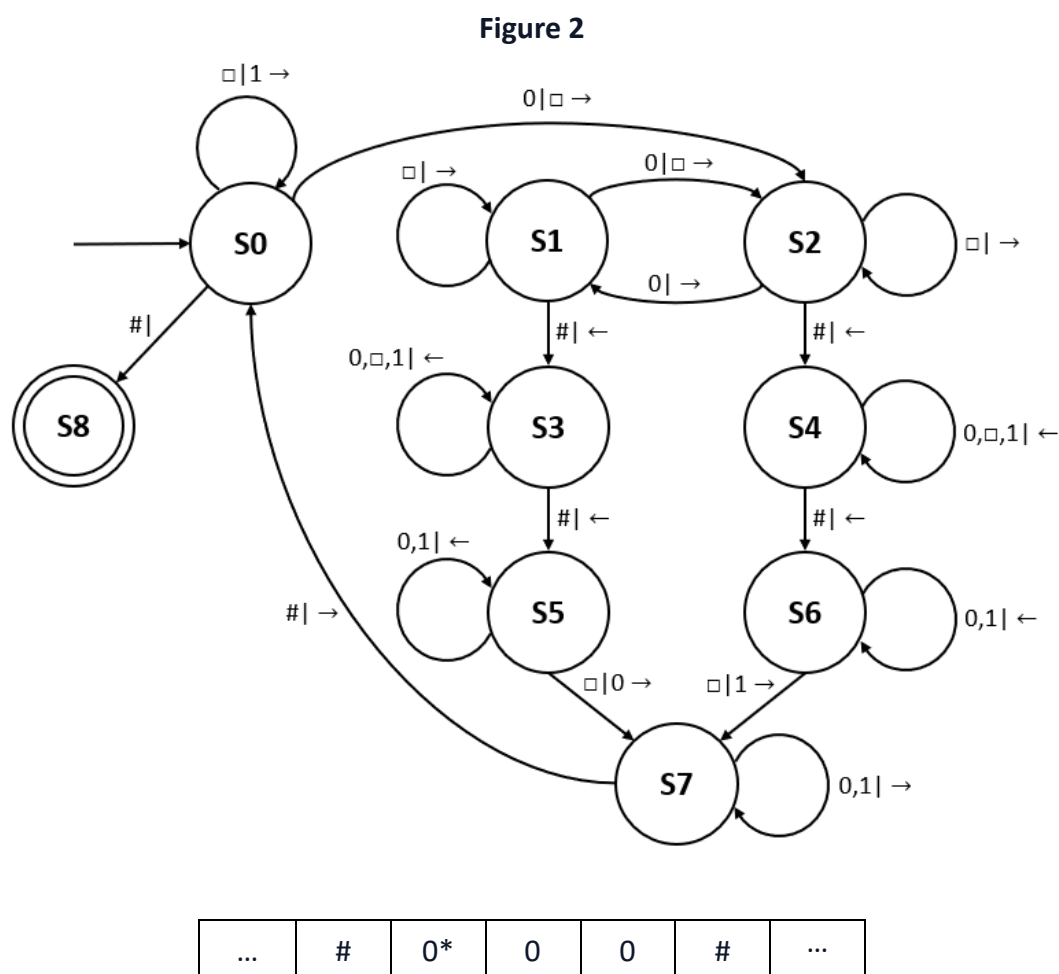
- c) What is the purpose of the transition from S5 to S6 in the Turing machine in **Figure 1**?

[1 mark]





- 3) **Figure 2** shows the transition function (represented as a state transition diagram) and part of the tape for a Turing machine designed to count the number of zeros between two # symbols on the tape. The current position of the read/write head is indicated by the * symbol.



The $\square$ symbol is used to denote an empty cell on the tape. The label on the transition from S6 to S7 means if the machine is in state S6 and a $\square$ is read from the tape, then a 0 should be written to the tape and then the read/write head is moved one cell to the right. The label on the transition from S3 to S3 means if the machine is in state S3 and a $\square$, a 0 or a 1 is read from the tape, then the read/write head is moved one cell to the left; the contents of the tape are not changed.

- a) Complete the unshaded cells of **Table 2** to show the result of tracing the computation of this Turing machine for the first 20 steps only. Show the contents of the tape, the current position of the read/write head and the current state as the input is processed. The top row of the table shows the initial state, the initial position of the read/write head and the starting contents of the tape.



Table 2

Tape

Current State

[illegible]

[5 marks]





b) Describe two circumstances when the transition function for this Turing machine will not correctly count the number of zeros between two # symbols on the tape. You should assume that:

- the read/write head starts immediately to the right of a cell containing a #
- there is a cell containing a # symbol somewhere on the tape to the right of the read/write head's initial position
- there are exactly two # symbols on the tape.

[2 marks]

