4.3.6 Optimisation Algorithms

1) **Figure 1** is a graph that shows the time it takes to travel between six cities labelled with the numbers 1 - 6.

When there is an edge between two nodes in the graph the edge is labelled with the time (in hours) it takes to travel between the two cities.

Figure 1

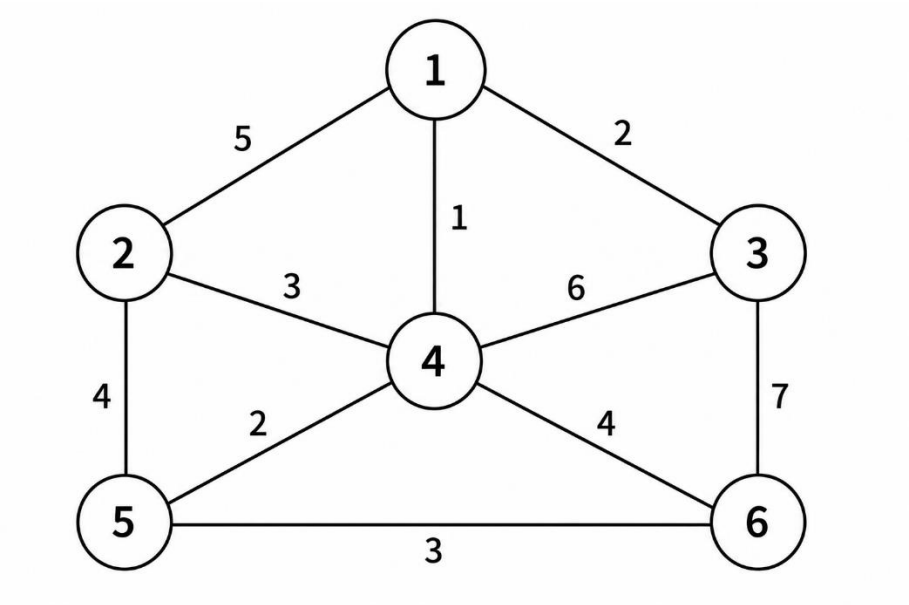

Figure 2 contains pseudo-code for a version of Dijkstra's algorithm used with the graph in **Figure 1**.

Q is a priority queue which stores nodes from the graph, maintained in an order based on the values in array D. The reordering of Q is performed automatically when a value in D is changed.

AM is the name given to the adjacency matrix for the graph represented in **Figure 1**.

Figure 2

```
Q ← empty queue
FOR C ← 1 TO 6
   D[C] ← 20
   P[C] ← -1
   ADD C TO Q
ENDFOR
D[1] ← 0
WHILE Q NOT EMPTY
   U ← get next node from Q
   remove U from Q
   FOR EACH V IN Q WHERE AM[U, V] > 0
      A ← D[U] + AM[U, V]
      IF A < D[V] THEN
         D[V] ← A
         P[V] ← U
      ENDIF
   ENDFOR
ENDFOR
OUTPUT D[6]
```

a) Complete the cells of **Table 1** to show the result of tracing the algorithm shown in **Figure 2**. Some of the trace, including the maintenance of Q, has already been completed for you.

Table 1

				D						P					
U	**Q**	**V**	**A**	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
-	1,2,3,4,5,6	-	-	20	20	20	20	20	20	-1	-1	-1	-1	-1	-1
				0											
1	2,3,4,5,6	2	**5**		**5**						**1**				
		3	**2**			**2**						**1**			
		4	**1**				**1**						**1**		
2	3,4,5,6	4	**8**												
		5	**9**					**9**						**2**	
3	4,5,6	4	**8**												
		6	**9**						**9**						**3**
4	5,6	5	**3**					**3**						**4**	
		6	**5**						**5**						**4**
5	6	6	**6**												
6	-														

[7 marks]

b) What does the output from the algorithm in Figure 2 represent?

[1 mark]

- **The shortest distance / time between locations/nodes 1 and 6;**

c) The contents of the array P were changed by the algorithm. What is the purpose of the array P?

[2 marks]

- **Stores the previous node in the shortest path to this node;**
- **Allows the path from node 1 to any other node to be recreated**

d) For each of the statements in **Table 2**, complete each row to indicate if the statement is true or false for Dijkstra's algorithm.

Table 2

	True or False?
Calculates the shortest path between a node and other nodes in a graph.	True
Can be used to prove that the Halting Problem cannot be solved.	False
Can be used with both directed and undirected graphs.	True
Can be used with both weighted and unweighted graphs.	False

[2 marks]