4.2.8 Vectors

Vectors

A vector is an ordered collection of numbers. They are a useful way to represent numeric data, points, directions, or quantities with multiple components.

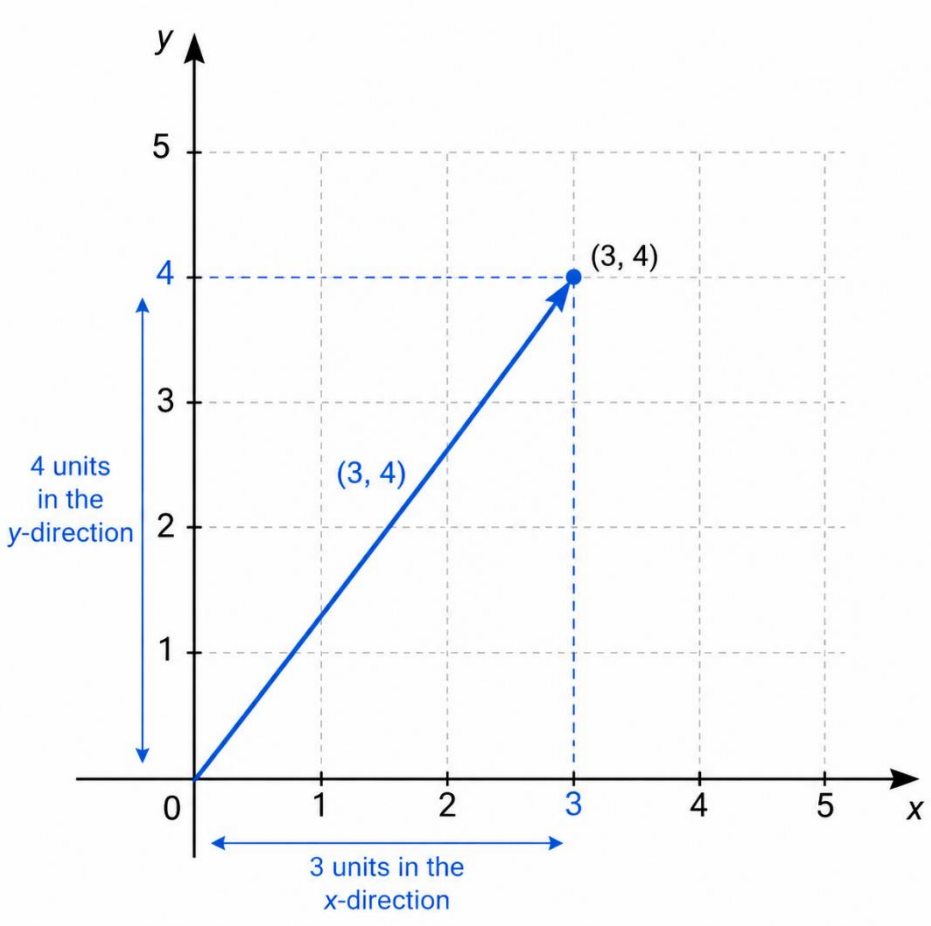
Example:

(3, 4) is a 2-vector

(2, -1, 7) is a 3-vector.

A 2D vector can be drawn as an arrow from the origin to a point.

For example, the vector (3, 4) can be visualised as moving 3 units along the x-axis and 4 units along the y-axis.



Vector operations

Addition

Add corresponding components of the vectors.

Worked example:

(1, 2) + (3, 4) = (4, 6)

Scalar Multiplication

Multiply every component by the same scalar value.

Worked example:

3(2, -1) = (6, -3)

Dot product

The dot product, also called the scalar product, takes two vectors of the same dimension and produces a single number. For vectors $a = (a_1, a_2, ..., a_n)$ and $b = (b_1, b_2, ..., b_n)$, the dot product is $a \cdot b = a_1b_1 + a_2b_2 + ... + a_nb_n$.

Worked example:

a = (2, 3, 1)

b = (4, -1, 5)

then, $a \cdot b = (2 \times 4) + (3 \times -1) + (1 \times 5) = 8 - 3 + 5 = 10$.

The dot product is used as part of the process of finding the angle between two vectors.

Convex combinations

A convex combination combines vectors using scalars that are non-negative and add up to 1. For example, $w = \alpha u + \beta v$ is a convex combination if $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta = 1$.

Convex combinations are useful because the resulting vector lies between the original vectors when visualised geometrically.

Example:

A good application would be **computer graphics / animation** where convex calculations can be used to calculate positions between two points, allowing objects to move smoothly between them.

Imagine a character moving between two positions:

- Position A: $u = (2,4)$
- Position B: $v = (6,8)$

A convex combination can calculate a position **between** A and B.

For example:

$$w = 0.25u + 0.75v$$

This gives:

$$w = 0.25(2, 4) + 0.75(6, 8)$$

$$w = (0.5, 1) + (4.5, 6)$$

$$w = (5, 7)$$

So (5, 7) is a position between the two original positions.